7th CCSS D2 Number System Conceptual Foundation (6-8 weeks)

Domain 2: The Number System 7.NS

D2 Cluster1: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts.
   c. Apply properties of operations as strategies to multiply and divide rational numbers.
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.

2.1.1 I can explain the solutions for operations on integers.e.g. I can explain why -2 times -3 is +6 or why -2 – (-6) is positive 4.

2.1.2 I can add and subtract natural and whole numbers, integers, fractions, and decimals, individually and combining more than one type of number (for example: 1/3 + 0.625 combines fraction and decimal).

2.1.3 I can multiply and divide natural and whole numbers, integers, fractions, and decimals, individually and combining more than one type of number (for example: $\frac{3}{2} \bullet (-2.2)$ combines fraction and negative decimals.

2.1.4 I can solve real-world problems involving all four operations on rational numbers.

2.1.5 I can apply the properties of operations" (commutative, associative, identity, distributive, and inverse properties, along with order of operations) to operations with rational numbers (for example 8 ÷ ¾ is (8 x 4) ÷ by 3 or (8 ÷3) x 4.

*Please note: The following conceptual foundation highlights important concepts and instructional support, but is not all inclusive. Please refer to "I Can" statements above for a more complete picture of content.

UNDERSTANDING OPERATIONS ON INTEGERS

Below we will discuss operation on integers. First we will look at addition/subtraction, then multiplication. The goal is that students are able to justify their solutions in operations. Students should have both a concrete understanding as well as an axiomatic understanding of operations.

ADDITION/SUBTRACTION Foundational Concept from CCSS 6th Grade Math:

1. **-a is the opposite of a. Students learn in 6th grade that a and –a are equal distances from 0 on a number line. Further they learn that a + -a = 0. Students should be comfortable with the idea, for example, that -6 is the opposite of 6 or 6 is the opposite of -6 and that 6 + -6 is 0.**

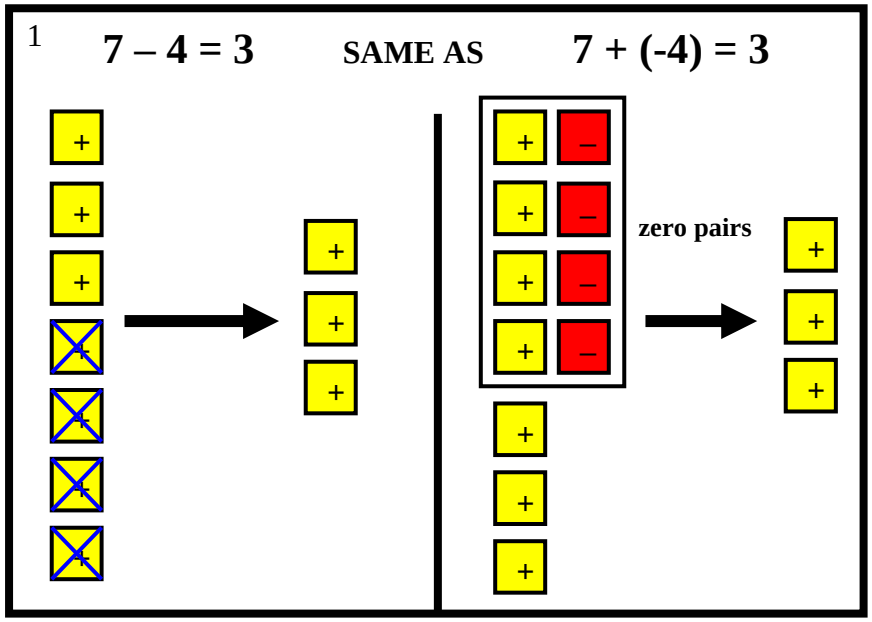
2. **MODELS: students should start their work with integers using concrete models. Models should then be tied to conceptualizations of the operations, and finally to automaticity (this likely means algorithms, though some students may visualize the meaning of the numbers and the operations for automaticity.)**

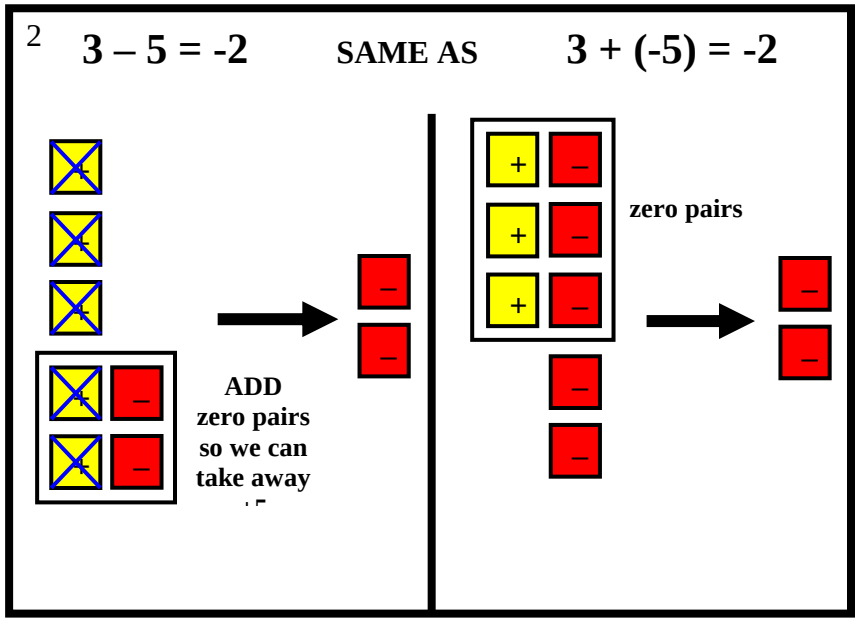
3. **Addition means the joining of sets.**

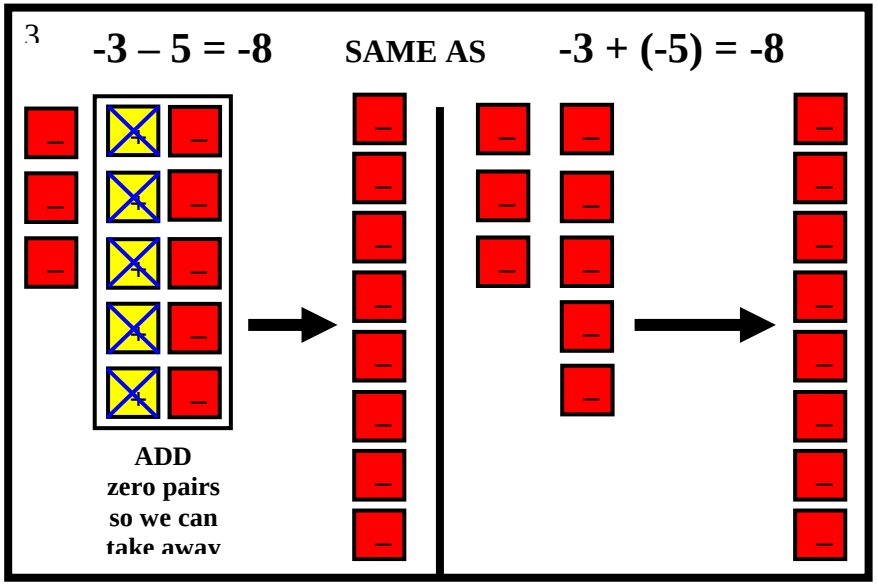
4. **Subtraction means:**
   a) **Take-away or**
   b) **Distance (between the two quantities)**

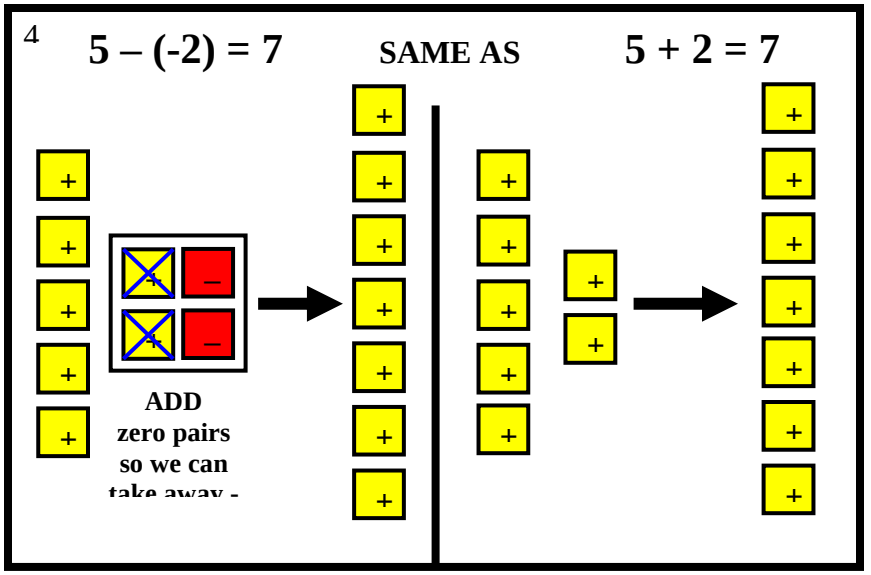
These concepts are at the foundation of all model representations of addition and subtraction:

Addition and Subtraction Chip Models

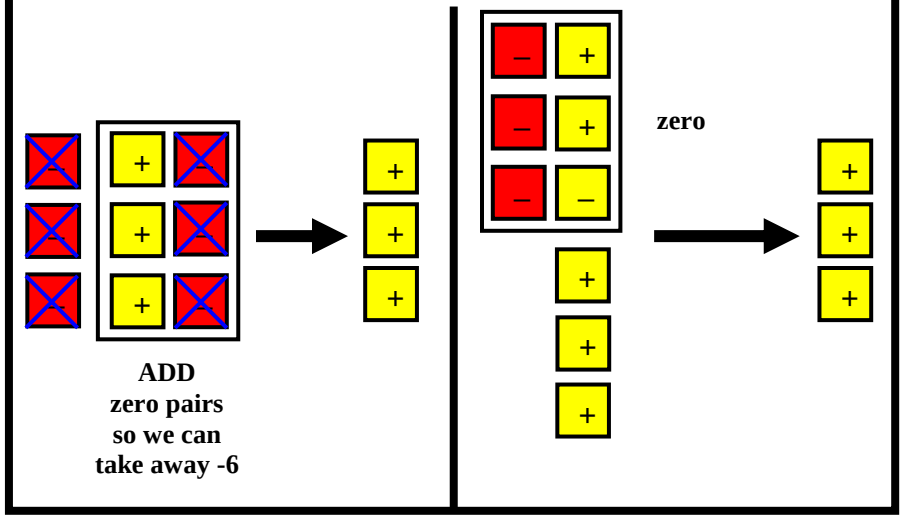








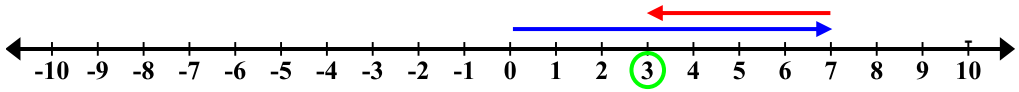
5. **-3 – (-6) = 3** **SAME AS** **-3 + 6 = 3**



Addition and Subtraction Number Line Models

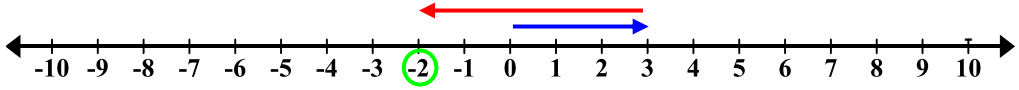
Number Line Model (Movement):

1. **7 – 4 = 3 OR 7 + (-4) = 3**



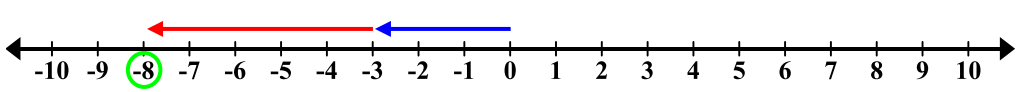
go forward 7
- go back 4

2. **3 – 5 = -2 OR 3 + (-5) = -2**



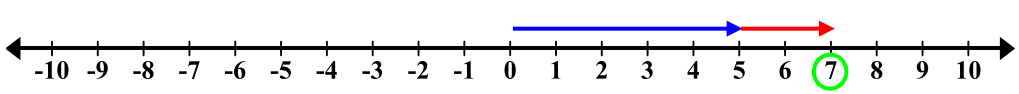
go forward 3
- go back 5

3. **-3 – 5 = -8 OR -3 + (-5) = -8**



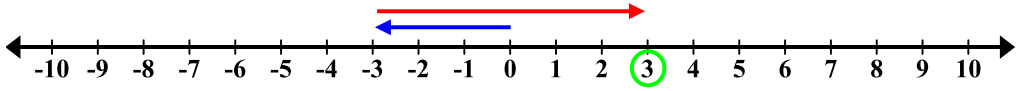
- go back 3
- go back 5 more

4. **5 – (-2) = 7 OR 5 + 2 = 7**



- go forward 5
- go the opposite of -2
or forward 2

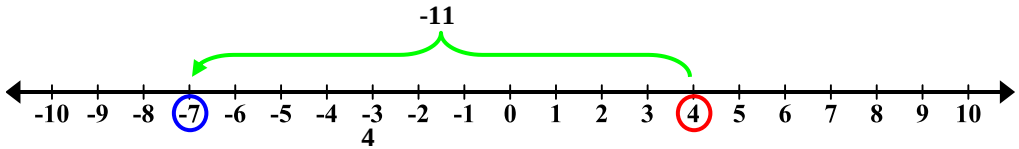
5. **-3 – (-6) = 3 OR -3 + 6 = 3**



- go back 3
- go the opposite of -6
or forward 6

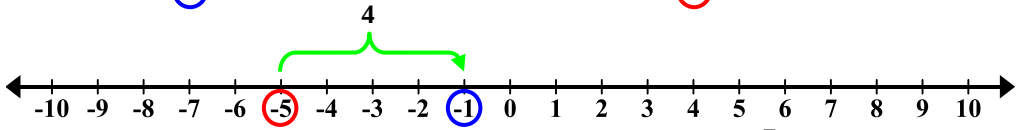
Number Line Model (Difference):

1. **-7 – 4 = -11**



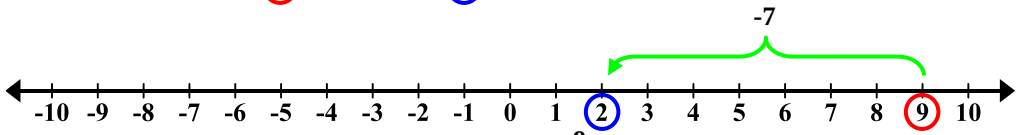
- find the difference between -7 and 4
- since -7 is to the left of 4 our result is -11

2. **-1 – (-5) = 4**



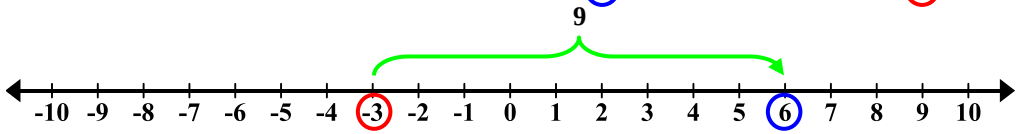
- find the difference between -1 and -5
- since -1 is to the right of -5 our result is 4

3. **2 – 9 = -7**



- find the difference between 2 and 9
- since 2 is to the left of 9 our result is -7

4. **6 – (-3) = 9**



- find the difference between -3 and 6
- since 6 is to the right of -3 our result is 9

Addition and Subtraction Real Life Model

1. 7 – 4 = 3 *or* 7 + -4 = 3
   a. start with **$7** — **7 – 4 = 3 *or* 7 + -4 = 3**
   b. spend **$4** — **7 – 4 = 3 *or* 7 + -4 = 3**
   c. end up with **$3** — **7 – 4 = 3 *or* 7 + -4 = 3**

2. 3 – 5 = -2 *or* 3 + (-5) = -2
   a. start with **$3** — **3 – 5 = -2 *or* 3 – 5 = -2**
   b. spend **$5** — **3 – 5 = -2 *or* 3 + -5 = -2**
   c. borrowed **$2** so **-$2** — **3 – 5 = -2 *or* 3 + -5 = -2**

3. -3 – 5 = -8 *or* -3 + (-5) = -8
   a. at **-$3** (in debt) — **-3 – 5 = -2 *or* 3 + (-5) = -8**
   b. spend **$5** more or **-$5** debt — **-3 – 5 = -8 *or* 3 + (-5) = -8**
   c. so total debt is **-$8** — **-3 – 5 = -8 *or* 3 + (-5) = -8**

4. 5 – (-2) = 7 *or* 5 + 2 = 7
   a. start with **$5** — **5 – (-2) = 7 *or* 5 – (-2) = 7**
   b. do the opposite of borrowing **$2** (earn **$2**) — **5 – (-2) = 7 *or* 5 + 2 = 7**
   c. end up with **$7** — **5 – (-2) = 7 *or* 5 + 2 = 7**

5. -3 – (-6) = 3 *or* -3 + 6 = 3
   a. at **-$3** (in debt) — **-3 – (-6) = 3 *or* -3 + 6 = 3**
   b. do the opposite of spending **$6** (earn **$6**) — **-3 – (-6) = 3 *or* -3 + 6 = 3**
   c. end up with **$3** — **-3 – (-6) = 3 *or* -3 + (-6) = 3**

Addition and Subtraction using properties:

"Properties of operations" includes commutative, associative, identity, distributive, and inverse properties; along with order of operations. **These properties should be shown and explained for ALL rational numbers.**

Set A	***Set B***	***Set C***	***Set D***
5 + 3 = _____ 3 + 5 = _____ These two problems give the same sum because of the **commutative property of addition**.	5 – 3 = _____ 3 – 5 = _____ These two problems give the same distance $\|2\|$ but opposite direction, 2 and -2. Subtraction finds the difference/distance between numbers. 5 – 3 = 2 because 5 is 2 units to the right of 3. 3 – 5 = -2 because 3 is 2 units to the left of 5. The **commutative property** does not hold for subtraction.	-5 + -3 = _____ -3 + -5 = _____ These two problems give the same sum because of the **commutative property of addition.**	-5 - -3 = _____ -3 - -5 = _____ These two problems give the same distance $\|2\|$, but opposite direction, 2 and -2. Subtraction finds the difference/distance between numbers. -5 – -3 = -2 because -5 is 2 units to the left of -3. -3 – -5 = 2 because -3 is 2 units to the right of 5. The **commutative property** does not hold for subtraction.

For more discussion on an axiomatic approach to addition and subtraction see:
http://2000clicks.com/MathHelp/AxiomsArithmetic.aspx

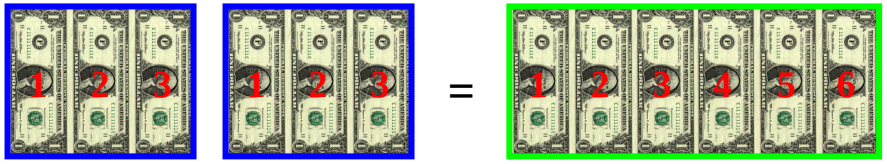
MULTIPLYING and DIVIDING INTEGERS

(For Division Models, work backwards from the multiplication models)

The goal is that students are able to justify their solutions in operations. Students should have both a concrete understanding as well as an axiomatic understanding of operations.

Concrete Money Model

1. $2 \cdot 3 = 6$ **2** days earn **$3** per day is $6: **2 • 3 = 6**



2. $2 \cdot (-3) = -6$ **2** days spent **$3** per day is -$6 or $6 debt: **2 • (-3) = -6**

3. $(-2) \cdot (3) = -(2) \cdot (3) = -6$ the opposite of (**2** days earning **$3** per day) is the opposite of **$6** OR **-$6** OR a debt of **$6** **-(2) • (3) = -6**

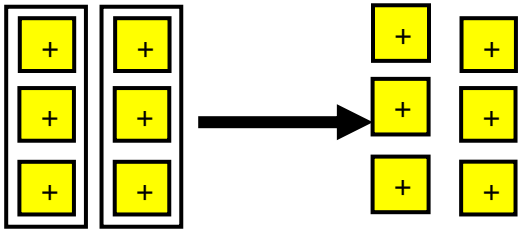
4. $(-2) \cdot (-3) = -(2) \cdot (-3) = 6$ the opposite of (**2** days spending **$3** per day) is the opposite of spending **$6** OR the opposite of **(-6)** OR 6 OR **-(2) • (-3) = 6**

Multiplication Group or Number Line Models:

1.

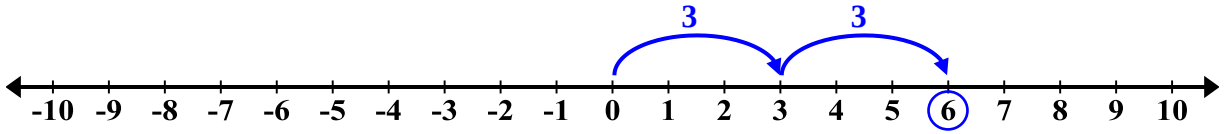
2 • 3 = 6

2 groups of 3



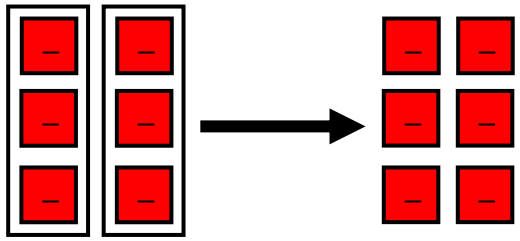
2 • 3 = 6

2 jumps of 3



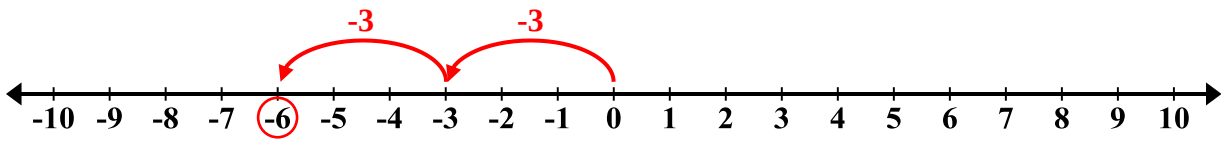
2. **2 • (-3) = -6**

2 groups of -3



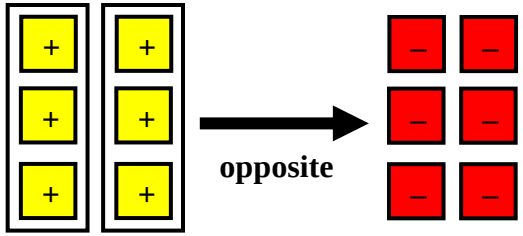
2 • (-3) = -6

2 jumps of -3



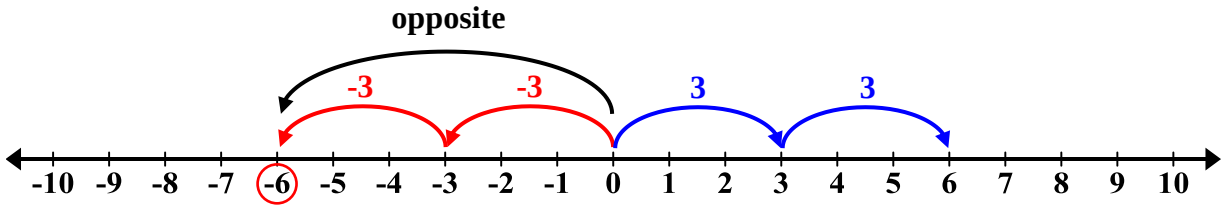
3. **(-2) • (3) = -(2 • 3) = -6**

opposite of 2 groups of 3



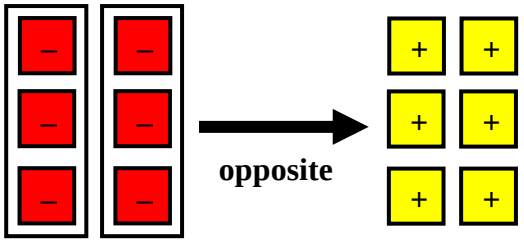
(-2) • (3) = -(2 • 3) = -6

opposite of 2 jumps of 3



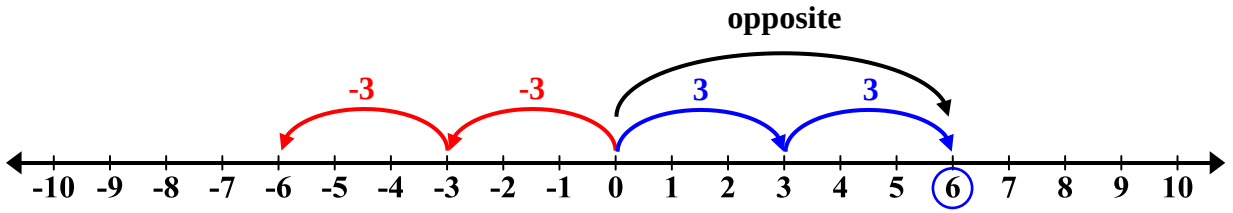
4. **(-2) • (-3) = -(2 • -3) = 6**

opposite of 2 groups of -3



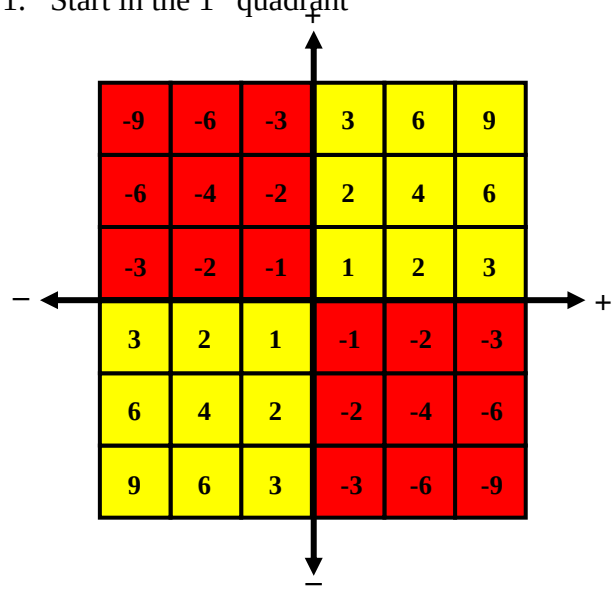
(-2) • (-3) = -(2 • -3) = 6

opposite of 2 jumps of -3



Multiplication Area Model:

1. Start in the 1st quadrant

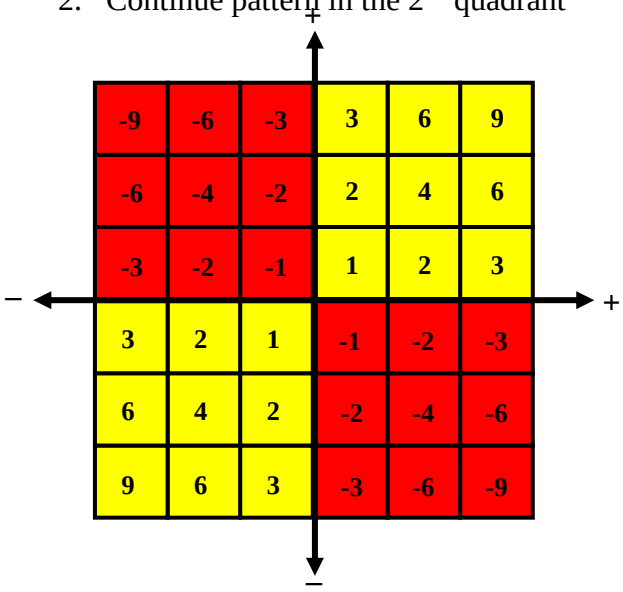


1 • 1 = 1
1 • 2 = 2
1 • 3 = 3

2 • 1 = 2
2 • 2 = 4
2 • 3 = 6

3 • 1 = 3
3 • 2 = 6
3 • 3 = 9

2. Continue pattern in the 2nd quadrant

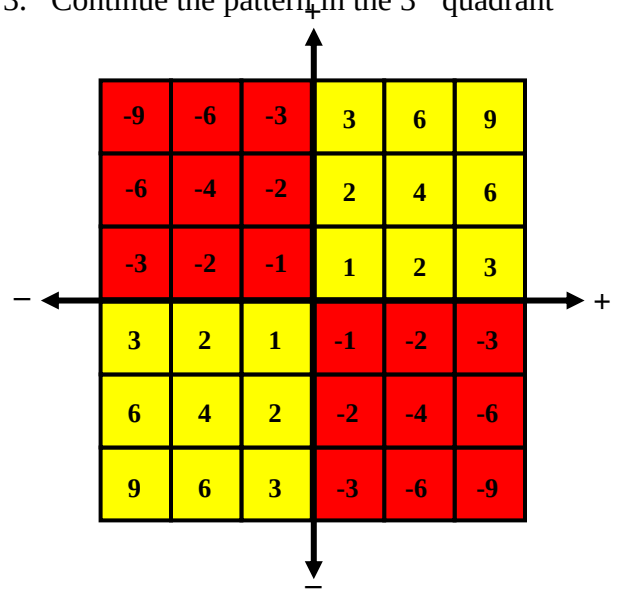


-1 • 1 = -1
-1 • 2 = -2
-1 • 3 = -3

-2 • 1 = -2
-2 • 2 = -4
-2 • 3 = -6

-3 • 1 = -3
-3 • 2 = -6
-3 • 3 = -9

3. Continue the pattern in the 3rd quadrant

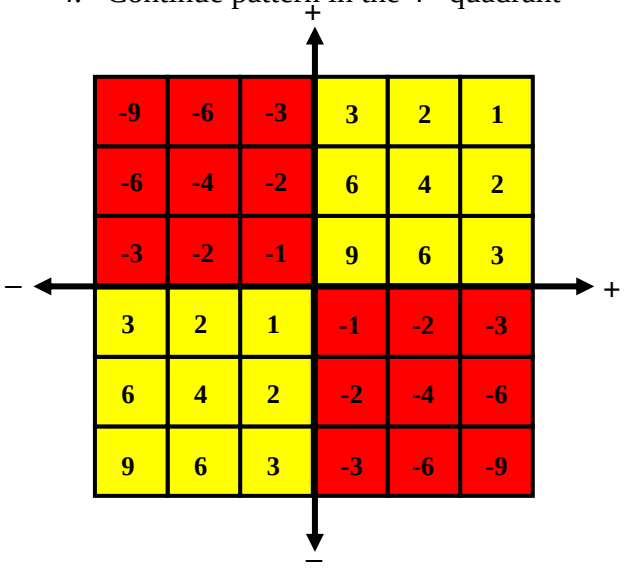


-1 • -1 = 1
-1 • -2 = 2
-1 • -3 = 3

-2 • -1 = 2
-2 • -2 = 4
-2 • -3 = 6

-3 • -1 = 3
-3 • -2 = 6
-3 • -3 = 9

4. Continue pattern in the 4th quadrant



1 • -1 = -1
1 • -2 = -2
1 • -3 = -3

2 • -1 = -2
2 • -2 = -4
2 • -3 = -6

3 • -1 = -3
3 • -2 = -6
3 • -3 = -9

Some students may find that approaching multiplication from patterning more intuitive:

Starting Number is 3	Multiply by 3	Using the Commutative Property
3	3 • 3 = 9	3 • 3 = 9
2	3 • 2 = 6	2 • 3 = 6
1	3 • 1 = 3	1 • 3 = 3
0	3 • 0 = 0	0 • 3 = 0
-1	3 • -1 = -3	-1 • 3 = -3
-2	3 • -2 = -6	-2 • 3 = -6
-3	3 • -3 = -9	-3 • 3 = -9

See Using Axioms:
http://2000clicks.com/MathHelp/AxiomsArithmetic.aspx