7th CCSS D3 Expressions and Equations Conceptual Foundation (6-8 weeks)

Domain 3: Expressions and Equations 7.EE

D3 Cluster 1: Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

D3 Cluster2: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
   For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
   For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

D3 Cluster 1: Use properties of operations to generate equivalent expressions.

3.1.1 I can add and subtract linear expressions with rational coefficients.
3.1.2 I can explain simplification of algebraic expressions (explain why 3x + x = 4x , but (3x)(x) is $3x^2$ OR why 3x + 2y cannot be simplified further but (3x) (2y) can be simplified.
3.1.3 I can draw representations for addition, subtraction, multiplication, and factoring of algebraic expressions and connect these drawings to symbolic representation.
3.1.4 I can factor and expand linear expressions with rational coefficients.
*3.1.5. I can restate expressions to make sense of real-life situations. (for example, the perimeter of a rectangle can be l+l+w+w or 2l+2w)
* ongoing

D3 Cluster2: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3.2.1 I can solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form, whole/fraction/decimal.
3.2.2 In multi-step real-life problems, I can convert between rational number forms (fractions, decimals, and percents) if appropriate.
3.2.3 In multi-step real-life problems, I can determine if and explain why an answer is reasonable using estimation and mental math.
3.2.4 I can solve a multi-step equation, including those using the distributive property.
3.2.5 I can solve a multi-step equation using real-life examples.
3.2.6 I can solve a multi-step inequality, graph the solution on a number line (including those using the distributive property).
3.2.7 I can solve a multi-step inequality using real-life examples and interpret the solution in the context of the problem.
*3.1.5. I can restate expressions to make sense of real-life situations. (for example, the perimeter of a rectangle can be l+l+w+w or 2l+2w)

Instructional models for helping students connect with algorithms

- **Bar model strategies for helping student correlate fractions, decimals and percent's.**
- **Models for helping students understand operations on variables.**
- **The relationship between the traditional multiplication algorithm and multiplication of polynomials.**
- **Models for solving algebraic equations.**
- **Inequality models, including models for why dividing by a negative "flips the sign around."**

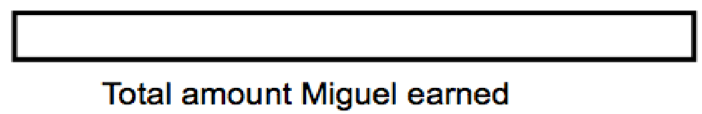
Solving problems with Fractions and Percent's: These types of problems should be solved pictorially before they are solved using algebraic equations. The same strategy can be applied to percent problems.

Example:

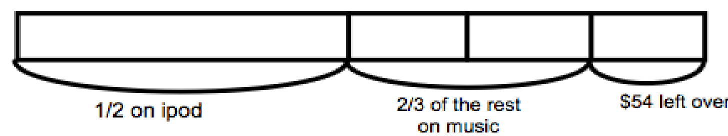
Miguel earned some money mowing lawns. He spent ½ of his money on an ipod and 2/3 of the rest on music for it. If he has $54 left, how much did he start with?

This problem can be easily solved using a model.

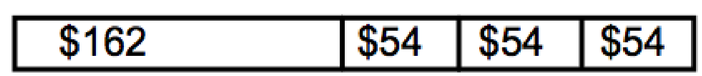
Start by drawing the total amount Miguel earned.



We know that ½ of the money was spent on and ipod and 2/3 of the rest on music. $54 was left over. See the picture.



So, we know that each of the other thirds of the half is $54, which means the other half must be $162 (or 3 time $54), hence the total amount he started with must have been $324.



Reasonable estimation and mental math

Example: This is a “naked” computation problem that students should be able to both estimate and find the correct answer without a calculator

5 x 1.98

Estimation:

Students should recognize that this problem is roughly 5x2 so the answer should be about 10. Further, they should recognize that the answer will be a little less then 10 because they are multiplying 5 by a number that is a little less than 2.

Mental Math

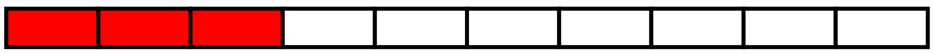
From above, a student should recognize that the answer close to 10 but a little smaller-- it is exactly 5 groups of .02 smaller. Hence it is .1 smaller, thus the answer is 9.9.

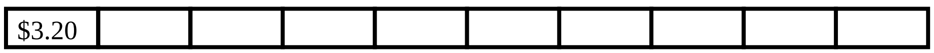
Estimating using Fraction-Decimal-Percentage equivalents (use bar models to represent problems)

Note: Please refer to the FDP equivalent worksheet provided below (last pages)

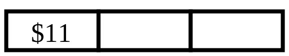
- **A 40% discount is the same as 2/5 of the total. Original cost = $25**

- **A 30% discount is the same as 3/10 of the total or close to 1/3 of the total**



Original cost $32 

(Each box would be worth $3.20. Subtract 3 boxes)  Total is__________

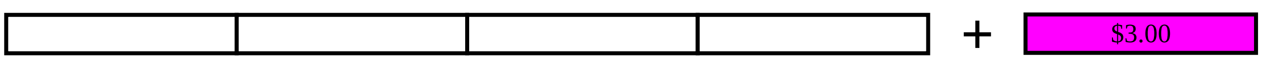
Original cost $33 

(Each box worth $11. Subtract 1 box.) [ ] Total is________

- **A 25% increase is the same as adding 1/4 of what you began.**

(Yesterday I earned $12. I earned 25% more today. How much?)

(Each box worth $3.00. Add another box. [ ] New total?_______)

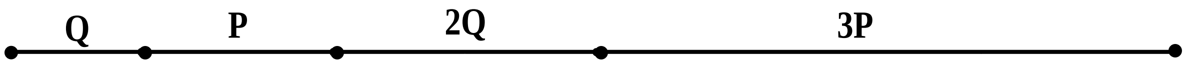


- **Using fractions decimals and percentages interchangeably in problems—use the easiest form.**

USING ALGEBRAIC PROPERTIES

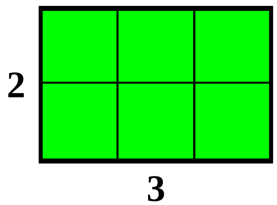
Models for addition and subtracting of variables (combining like terms).

- Picture model ☼☼☼ + ✈✈ + ☼☼☼☼ = 7☼ + 2✈
- Linear model

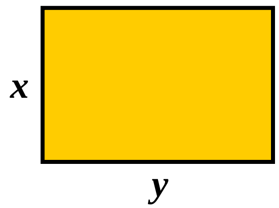


The length of the line is **q + p + 2q + 3p = 3q + 4p** in length.

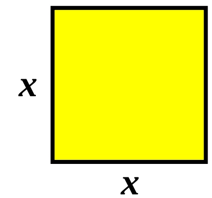
Models for multiplication of variables



$(2)(3) = 6\, sq\, units$



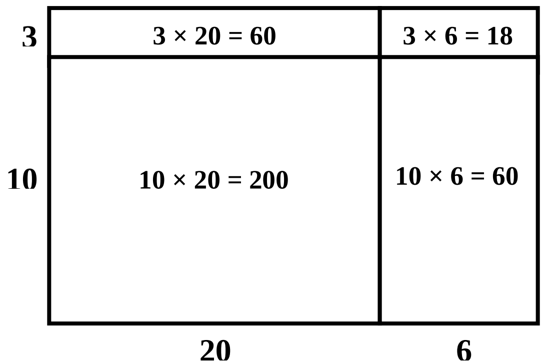
$(x)(y) = xy$



$(x)(x) = x^2$

Models for the distributive property/factoring

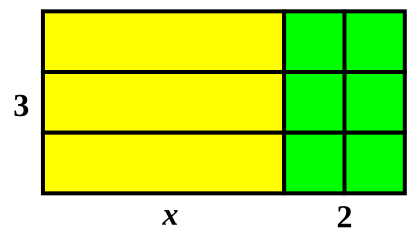
$$\begin{array}{r} 13 \\ \times\ 26 \\ \hline 78 \\ 260 \\ \hline 338 \end{array}$$



Examine the distributive shown above.

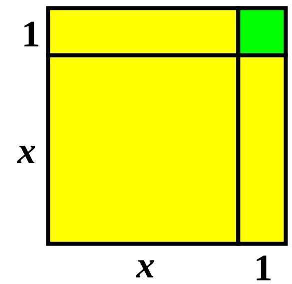
13 × 6 = 6(10 + 3) or 60 + 18

13 × 20 = 20(10 + 3) or 200 + 60



…and now with symbols

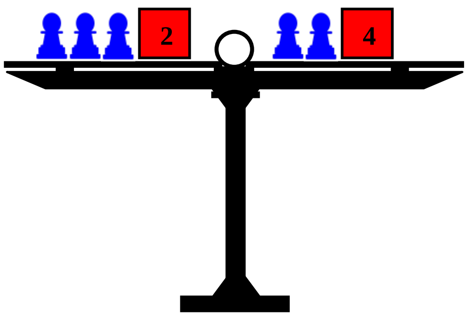
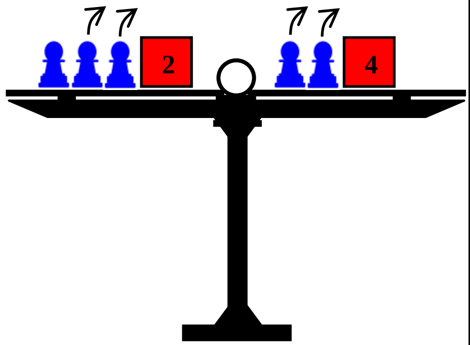
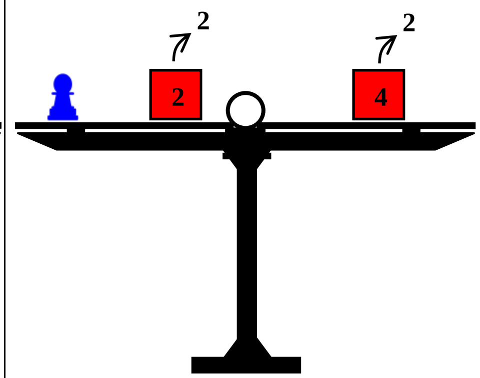
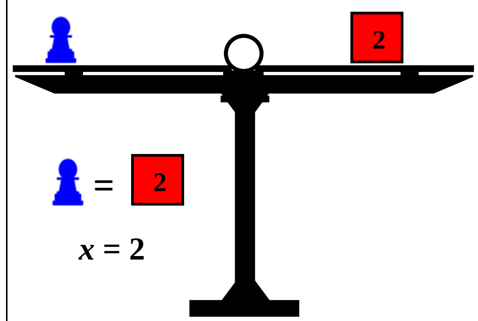
$3(x+2) = 3x+6$



$(x+1)(x+1) = x^2 + 2x + 1$

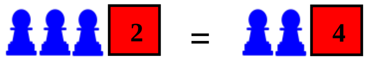
“Hands-On Equations” MODEL for SOLVING EQUATIONS

Solve the equation 3*x* + 2 = *x* + 4:

Step 1: Set up the equation.	***Step 2:*** Remove (subtract) two blue pawns from each side of the scale.	***Step 3:*** Remove (subtract) a value of 2 from each side of the scale.	***Step 4:*** A blue pawn has a value of 2.
			

$x = 2$

Step 5: Check your solution by substituting 2 back into the original set up.



2 + 2 + 2 + 2 = 2 + 2 + 4

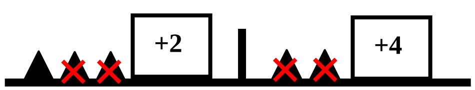
8 = 8

x **= 2** is the correction solution.

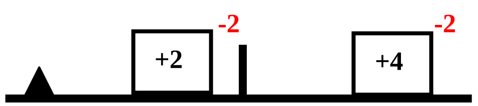
Students simultaneously draw the model (from above) and solve algebraically:

Step 1:

Step 2:



Step 3:

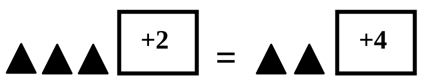


Step 4:

Algebraically:

$$\begin{array}{rcr} 3x+2 &=& 2x+4 \\ -2x & & -2x \\ \hline 1x+2 &=& 4 \\ -2 & & -2 \\ \hline x &=& 2 \end{array}$$

Step 5: Check your solution.



2 + 2 + 2 + 2 = 2 + 2 + 4

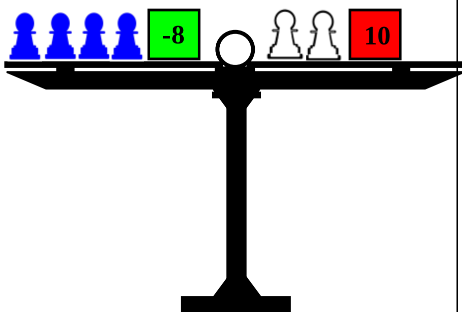
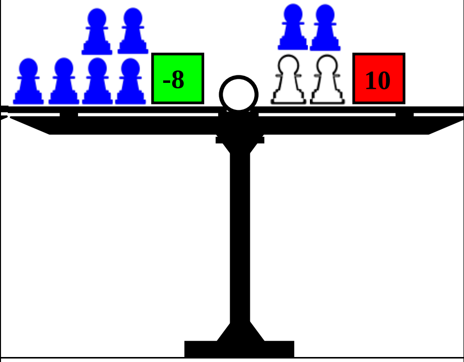
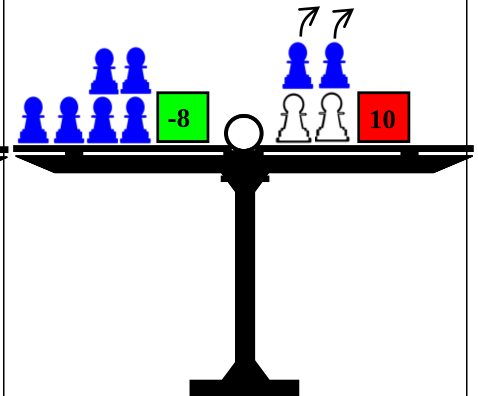
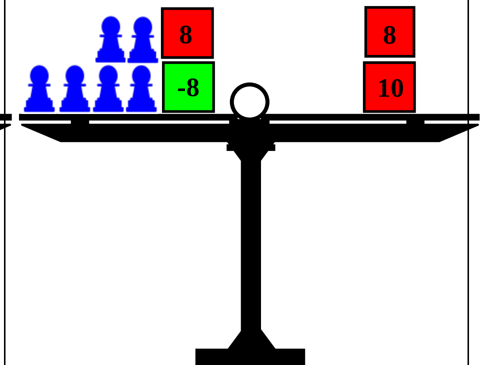
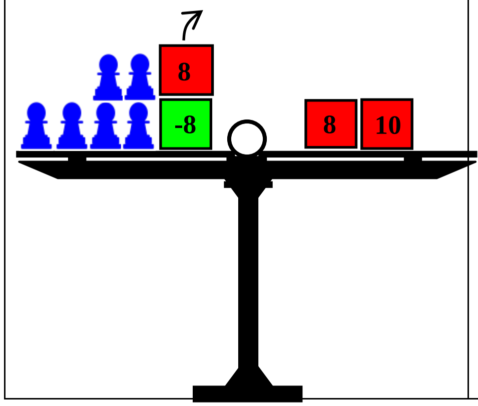
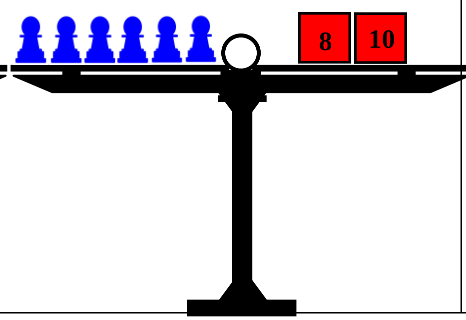
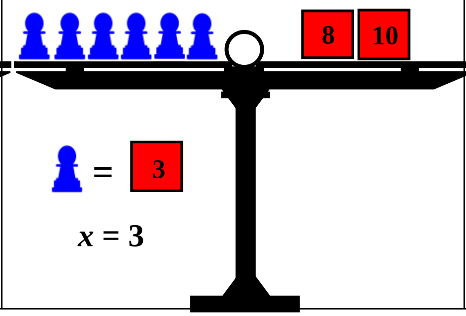
8 = 8

$$3 \cdot 2 + 2 = 2 \cdot 2 + 4$$

$$6 + 2 = 4 + 4$$

8 = 8

Solve the equation $4x - 6 = -2x + 10$:

Step 1: Set up the equation.	***Step 2:*** Add two blue pawns to each side of the scale to create zero pairs.	***Step 3:*** Remove the zero pairs from the left side of the scale.	***Step 4:*** Add 8 to each side of the scale to create a zero pair.
			
Step 5: Remove the zero pair from the right side of the scale.	***Step 6:*** Divide 18 evenly among each of the 6 blue pawns.	***Step 7:*** Each blue pawn has a value of 3.	***Step 8:*** Check your solution by substituting 3 back into the original set up.
		 **$x = 3$**	 =  **3 + 3 + 3 + 3 – 8 = -3 + -3 + 10** **4 = 4** ***x* = 3** is the correction solution.

Students might benefit from thinking about solving equations as "backwards" Order of Operations

BACKWARDS

$$10 = 2x + 4$$

steps (what was done to *x*)	undo	*x* = 3
• 2	÷ 2	6
+ 4	- 4	10

$$\frac{-3x+4}{2} = 11$$

steps (what was done to *x*)	undo	*x* = -6
• (-3)	÷ (-3)	18
+ 4	- 4	22
÷ 2	• 2	11

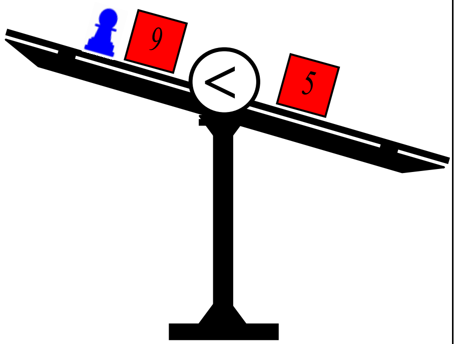
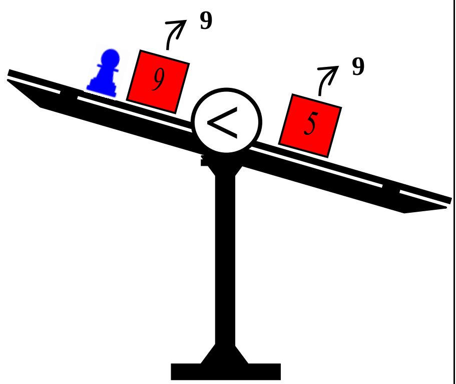
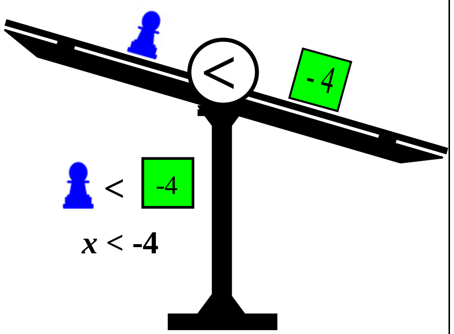
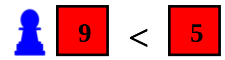
BALANCE

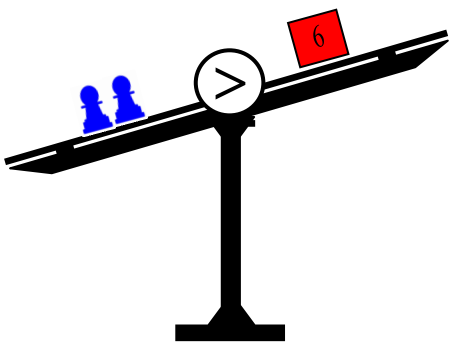
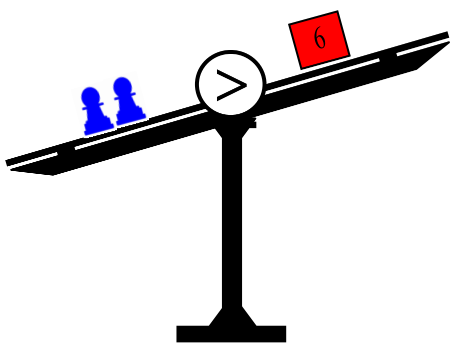
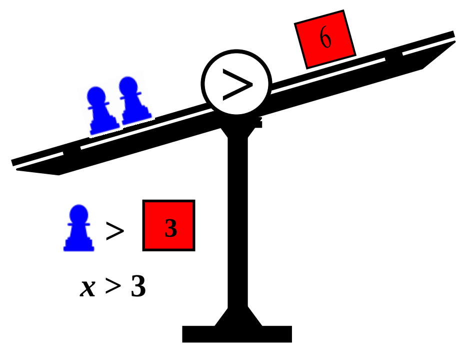
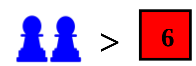
$$10 = 2x + 4$$
$$-4 \qquad -4$$
$$\frac{6}{2} = \frac{2x}{2}$$
$$3 = x$$

$$2 \cdot \frac{-3x+4}{2} = 11 \cdot 2$$
$$-3x + 4 = 22$$
$$-4 \quad -4$$
$$\frac{-3x}{-3} = \frac{18}{-3}$$
$$x = -6$$

Solving INEQULITIES Models

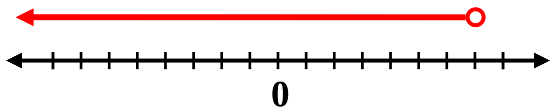
Solve the inequality $x + 9 < 5$:

Step 1: Set up the inequality.	***Step 2:*** Remove (subtract) a value of 9 from each side of the scale.	***Step 3:*** Each pawn has a value less than -4.	***Step 4:*** Check your solution by substituting a value for the pawn that is less than -4.
		 **$x < -4$**	Since -7 is less than -4, let's substitute it for our blue pawn.  **-7 + 9 < 5** **2 < 5** **$x < -4$** is the correction solution.

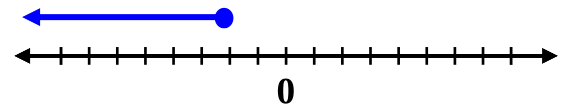
Step 1: Set up the inequality.	***Step 2:*** Divide 6 evenly among each blue pawn.	***Step 3:*** Each pawn has a value greater than 3.	***Step 4:*** Check your solution by substituting a value for each pawn that is greater than 3.
		 **$x > 3$**	Since 5 is greater than 3, let's substitute it for our blue pawns.  **5 + 5 > 6** **10 > 6** **$x > 3$** is the correction solution.

Graphing Inequalities on a Number Line:

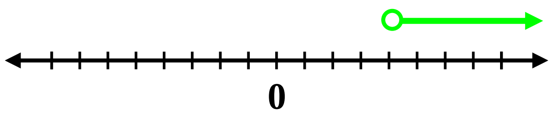
The following are examples of graphing the inequalities <, ≤, >, and ≥ on a number line.



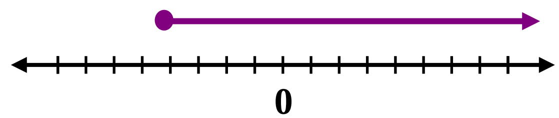
$$x < 7$$



$$x \leq -2$$



$$x > 4$$



$$x \geq -4$$

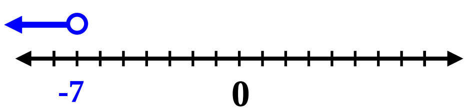
Solve and graph the following inequalities.

1. $2b + 4 < -10$

$\quad -4 \quad -4$

$\frac{2b}{2} < \frac{-14}{2}$

$b < -7$

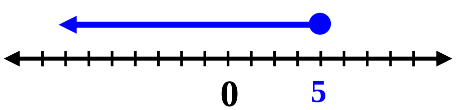


2. $12 \leq 3b - 3$

$+3 \quad +3$

$\frac{15}{3} \leq \frac{3b}{3}$

$5 \leq b$

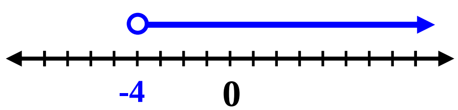


3. $-b - 6 < -2$

$\quad +6 \quad +6$

$\frac{-b}{-1} > \frac{4}{-1}$ *divide by −, change the sign*

$b > -4$



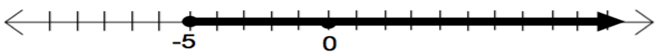
In solving Inequalities, explain why dividing by a negative reverses the direction of the inequality sign.

Students often struggle with the concept that division by a negative reverses the direction of the inequality sign. Let's look of some examples to show why this works.

Example 1:

We know that 2 is most certainly less than 3, hence the statement **2<3** is most a true statement. But if you multiply the inequality by -1, you get -2<-3. This inequality is saying that -2 smaller than -3, but this is a FALSE statement.

Any positive number gets larger as the absolute value of that positive number gets larger, but any negative number gets SMALLER as the absolute value of that number gets LARGER. This makes working with inequalities and multiplying by negatives tricky. Think about it in terms of sets of numbers, for example x≥-5 (see below). This is the set of all numbers that are larger than -5.



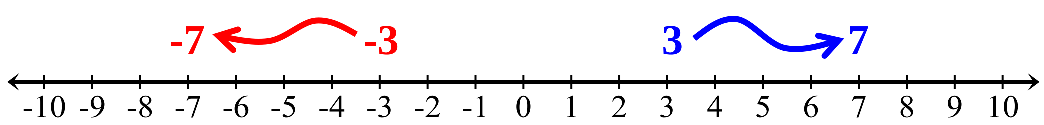
Now let's explore with this statement. Suppose I want to add 3 to both sides of this inequality: x+3≥-5 +3 or x+3≥-2, will the same set of numbers as above make this statement true? Yes. If I substitute -4 in for x the statement will be true, but if I substitute -6 it will not. I can do the same thing buy trying to subtract 3 from both sides, or multiplying by any positive number. But let's try simply multiplying both sides by -1: -x≥5 would be the new statement, is this statement still true for the values above, let's again check -4 and -6. If I substitute -4 into the new inequality, I get 4≥5 NOT true, this is a contradiction to the original inequality. If I substitute -6, I get a true statement which is also a contradiction.

Example 2:

When you multiply or divide by a **negative number** you have to **reverse** the inequality. **Why?**

Well, just look at the number line!

For example, from 3 to 7 is **an increase**, but from -3 to -7 is **a decrease.** See how the inequality sign reverses (from < to >)?



-7 < -3 **7 > 3**

Solve: -2*y* < -8

Divide both sides by -2 ... and **reverse the inequality**!

$$\frac{-2y}{-2} < \frac{-8}{-2}$$

$$y > 4$$

And that is the correct solution: ***y* > 4**

(Note that I reversed the inequality **on the same line** I divided by the negative number.)

So, just remember:

When multiplying or dividing by a negative number, **reverse** the inequality

Problem Solving with Inequalities:

1. Alex earns $7.50 per hour by working after school. Alex needs at least $60 to buy a video game. Write and solve an inequality that shows how many hours he must work to buy the video game.

$$\frac{7.50x}{7.50} \geq \frac{60}{7.50} \qquad x \geq 8 \textit{ hours}$$

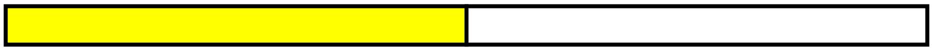
2. To make a profit, Elwin's Electronics has to sell more than 200 items in a month. If Elwin sold 30 items in the first week of the month, write and solve an inequality that shows how many items need to be sold in the remaining weeks to earn a profit.

$$\begin{array}{l} x + 30 > 200 \\ \underline{\;\;\;-30 \quad -30} \\ x \qquad > 170 \textit{ items} \end{array}$$

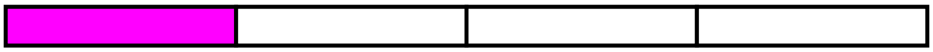
SOLVE PROBLEMS and ESTIMATE SOLUTIONS using RATIONAL NUMBERS and EQUIVALENTS

Models Fractions/Decimals/Percent equivalents:

1. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)



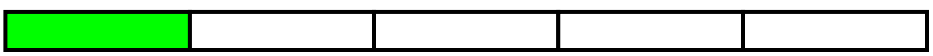
2. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)
   Two parts _______ _______ _______



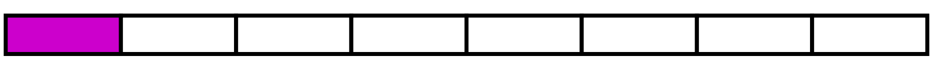
3. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)
   Two parts _______ _______ _______

   Three parts _______ _______ _______

4. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)
   Two parts _______ _______ _______
   Three parts _______ _______ _______
   Four parts _______ _______ _______
   Five parts _______ _______ _______



5. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)
   Two parts _______ _______ _______
   Three parts _______ _______ _______
   Four parts _______ _______ _______
   Five parts _______ _______ _______
   Six parts _______ _______ _______
   Seven parts _______ _______ _______
   Eight parts _______ _______ _______



6. One part below is worth _______ (fraction) = _______ (decimal) = _______ (percentage)
   Two parts _______ _______ _______
   Three parts _______ _______ _______
   Four parts _______ _______ _______
   Five parts _______ _______ _______
   Six parts _______ _______ _______
   Seven parts _______ _______ _______
   Eight parts _______ _______ _______
   Nine parts _______ _______ _______