8^{th} CCSS D4 Geometry Conceptual Foundation (6-8 weeks)

Domain 4: Geometry 8.G **D4 Cluster 1: Understand congruence and similarity using physical models, transparencies, or geometry software.** 1. Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. Function notation is not required in Grade 8. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. 5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
4.1.1 I can use the properties of translations, rotations, and reflections on line segments, angles, parallel lines or geometric figures. 4.1.2 I can show and explain two figures are congruent using transformations (explaining the series of transformations used). 4.1.3 I can determine the new coordinate of a figure given a dilation, translation, rotation, or reflection. 4.1.4 I can describe transformations and/or dilations that produce similar figures and explain similarity of figures in terms of dilation and/or transformation.. 4.1.5 I can show triangles are similar by AA and explain why AA is enough to show similarity. 4.1.6 I can show and/or explain how the angle-sum and exterior-angle theorems of a triangle are true. 4.1.7 I can identify angle pairs created by parallel lines cut by a transversal and explain which angle pairs are congruent or supplementary and why.
D4 Cluster 2: Understand and apply the Pythagorean Theorem. 6. Explain a proof of the Pythagorean Theorem and its converse. 7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
4.2.1 I can give or explain a proof of the Pythagorean Theorem and its converse (prove perpendicular sides or right triangle). 4.2.2 I can apply the Pythagorean Theorem in real-world situations or drawings to find unknown side lengths in right triangles in two and three dimensions. 4.2.3 I can use the Pythagorean Theorem to find the distance between two points on a coordinate system. (I can derive the distance formula—Honors.) 4.2.4 I can describe patterns in special right triangles. (Honors)
D4 Cluster 3: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
4.3.1 I know and use the formulas for volumes to solve real world and mathematical problems involving cones, cylinders, and spheres

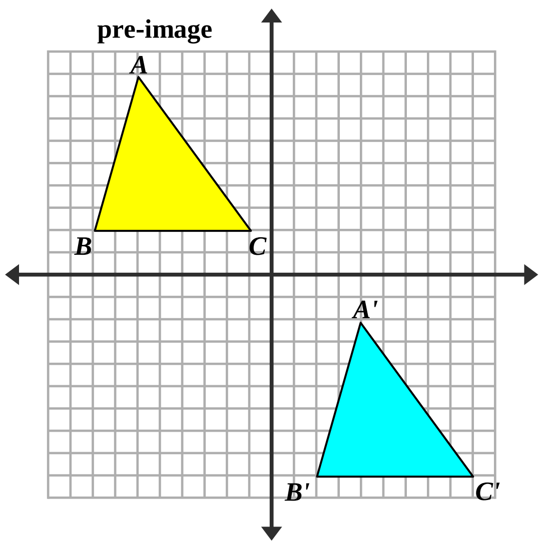
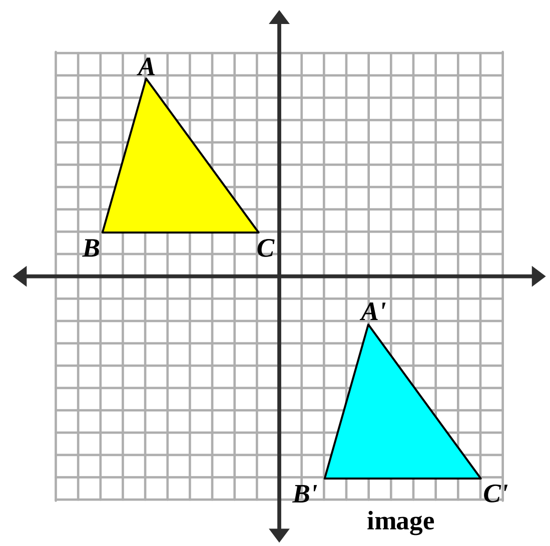
For further conceptual foundation support (beyond what's included below), please refer to "I Can" statements, the "8 D4 Geometry" assessment, and corresponding answer sheet.

TRANSFORMATIONS

Students should be able to apply transformations to different objects. They should describe transformations as functions that take points in the plane as inputs and give other points as outputs – making connections to functions and the transformations they have done on functions. Students should also make comparisons between transformations that preserve distance and angle to those that do not. They should be able to predict whether or not a transformation will maintain the congruence between the pre-image and image – i.e. the input object is congruent to the output object.

<u>NOTE</u>: Using software that allows students to manipulate objects and perform these transformations is highly recommended. For example: Geometer's Sketchpad, Geogebra and other programs. These often come with materials, diagrams that the software company has already created to facilitate explorations of these topics.

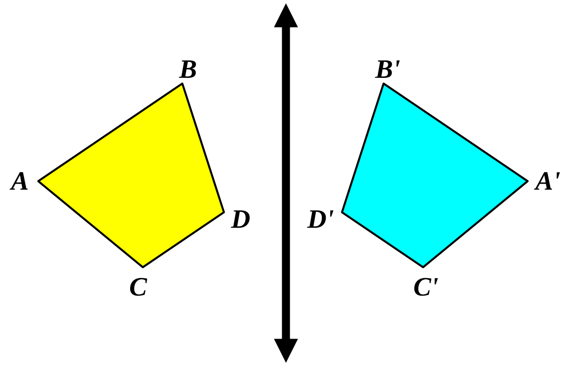
Transformation Terminology

***pre-image*:** original figure before transformation; <u>input</u> into the transformation function 	***image*:** figure after transformation; <u>output</u> from the transformation function 

TRANSFORMATIONS (rigid motion transformations)

***Rigid Motion Transformations*:** preserve congruence

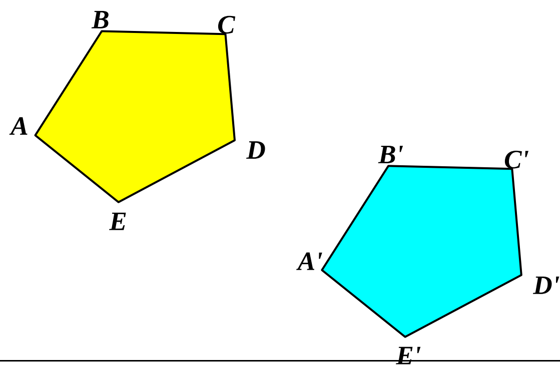
- **reflection** – flip
  - The reflection of an object is called its ***image***. If the original object (which is called the ***pre-image***) was labeled with letters, such as polygon ***ABCD***, the image may be labeled with the same letters followed by a ***prime*** symbol, ***A'B'C'D'***.



  - The line (where a mirror may be placed) is called the ***line of reflection***. The distance from a point to the line of reflection is the same as the distance from the point's image to the line of reflection.
  - A reflection can be thought of as folding and "flipping" an object over the line of reflection.
  - Connecting this to functions – the pre-image would be the input to the transformational function and the image would be the output.
  - Connect this to the idea of absolute value and distance from some origin.

- **translation** – slide
  - A ***translation*** "slides" an object a fixed distance in a specified direction. The original object and its translation have the **same shape and size**, and they **face in the same direction**, they are oriented the same way. The word "translate" in Latin means "carried across".
  - If the pre-image (input was polygon **ABCDE**, then the image after a translation has taken place would be polygon ***A'B'C'D'E'.***
  - A translation is strictly additive—only adding values, as opposed to dilating (scale or multiplication) an object—this would not preserve congruence.

Think of polygon ***ABCDE*** as sliding two units to the right and one unit down. Its new position is labeled ***A'B'C'D'E'.***

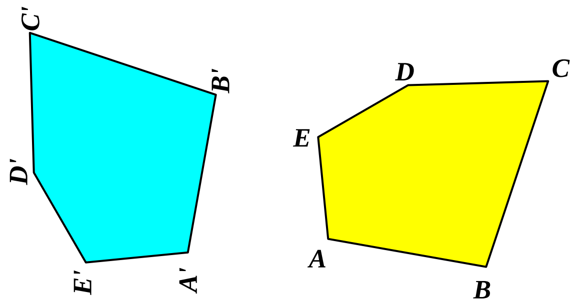


Think of polygon ***ABCDE*** as sliding two units to the right and one unit down. Its new position is labeled ***A'B'C'D'E'.***

TRANSFORMATIONS (rigid motion transformations - continued)

***Rigid Motion Transformations*:** preserve congruence

- **rotation** – turn
  - A rotation turns an object around a point. Rotations can occur in either a ***clockwise*** (to the right) or ***counterclockwise*** (to the left) direction.



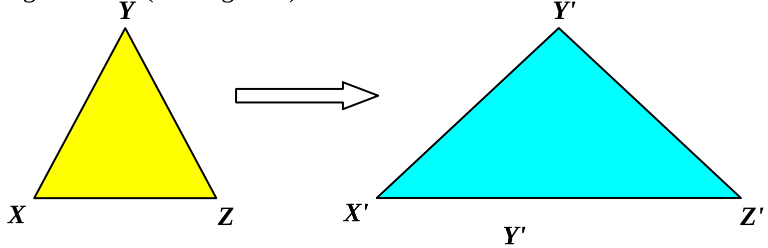
This rotation of the pre-image polygon ***ABCDE*** is 90° counterclockwise. The output is the image ***A'B'C'D'E'.***

- A **positive angle** of rotation turns the figure **counterclockwise**, and a **negative angle** of rotation turns the figure in a **clockwise direction**.
- Notice that a rotation does not change the size of the figure.

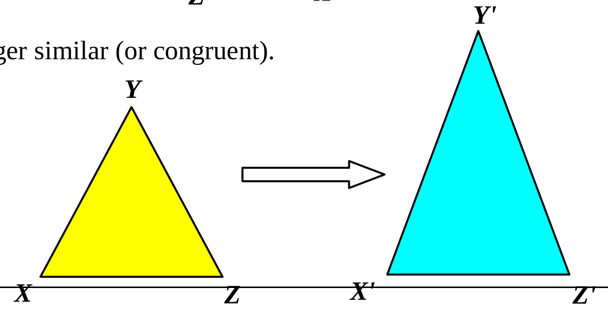
TRANSFORMATIONS (non-rigid motion transformations)

***Non-Rigid Transformations*:** do not preserve congruence

- **dilation** – A dilation is a multiplicative stretch which produces a similar figure. The stretch is the same both horizontally and vertically. The amount by which a figure grows or shrinks is called the scale factor.
- **horizontal stretch** – a stretch in which a plane figure is distorted horizontally.
  - The size and shape of the figure is changed so the image and pre-image are no longer similar (or congruent).



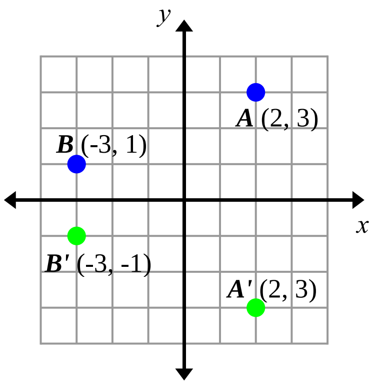
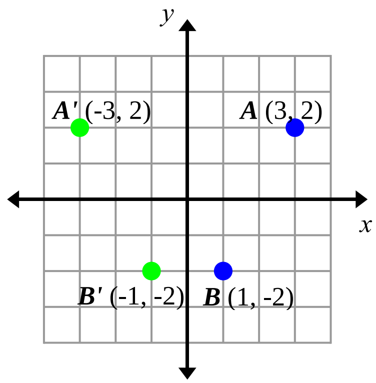
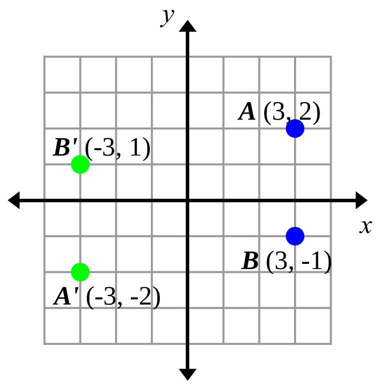
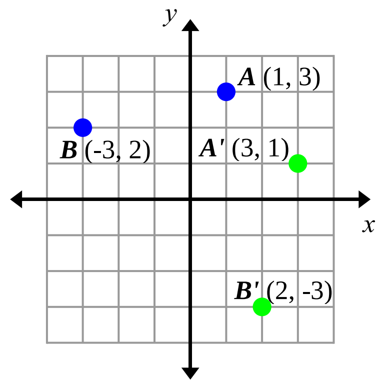
- **vertical stretch** – a stretch in which a plane figure is distorted vertically.
  - The size and shape of the figure is changed so the image and pre-image are no longer similar (or congruent).



EXAMPLE (translation in the coordinate plane)

NOTE: reflection in the origin is the same thing as a 180° rotation about the origin.

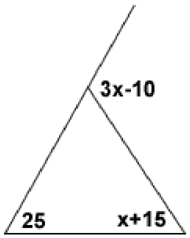
Reflections in the Coordinate Plane

Reflection	*x*-axis	*y*-axis	origin	*y* = *x*
Pre-image to Image	(*a*, *b*) → (*a*, -*b*)	(*a*, *b*) → (-*a*, *b*)	(*a*, *b*) → (-*a*, -*b*)	(*a*, *b*) → (*b*, *a*)
How to find coordinates	Multiply the *y*-coordinate by -1.	Multiply the *x*-coordinate by -1.	Multiply both coordinates by -1.	Interchange the *x*- and *y*-coordinates.
Example				

ANGLE THEOREMS—Show/explain/identify, PYTHAGOREAN THEOREM and converse—prove/explain/apply

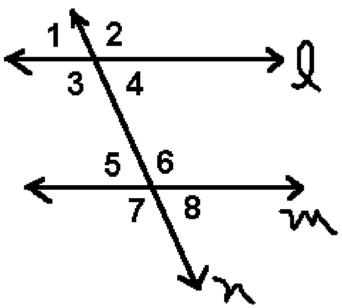
Triangle Sum and Exterior Angle Theorems

- The interior angles of a triangle sum to 180 degrees
- The measure of an exterior angle of a triangle = the sum of the measures of the two non-adjacent interior angles of the triangle.
- The measure of an exterior angle of a triangle is greater than either of its two non-adjacent interior angles.
- Sample problem: If the measure of the exterior angle = (3x - 10) degrees and the measure of the two remote interiors angles are 25 degrees and (x + 15) degrees, find x.



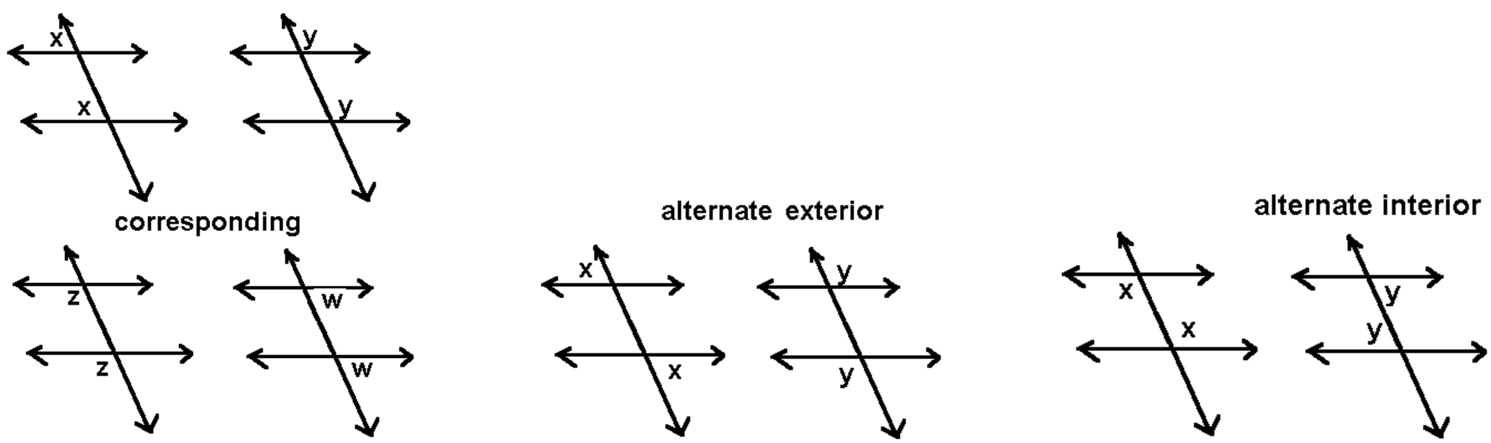
Angles formed by Parallel Lines with a Transversal

NOTE: It is recommended that students use geometry software, Geometer's Sketchpad or Geogebra (online), create parallel lines with transversals, measure the angles and derive or prove the following:



- Vertical angle pairs are congruent, for example angles 1 and 4.
- Adjacent angle pairs are congruent, for example angles 5 and 6.

- Corresponding angles, alternate exterior angles, and alternate interior angles are congruent (see examples below).



After students are fluent with congruent angle pairs, they can solve problems using congruent and/or supplementary angle pairs. This website summarizes the angle pair theorems and then gives problems for students.

http://www.mathsisfun.com/geometry/parallel-lines.html

Pythagorean Theorem

Some background: The Pythagorean Theorem was one of the earliest theorems known to ancient civilizations. This famous theorem is named for the Greek mathematician and philosopher, Pythagoras. Pythagoras founded the Pythagorean School of Mathematics in Cortona, a Greek seaport in Southern Italy. The Pythagorean Theorem is Pythagoras' most famous mathematical contribution. According to legend, Pythagoras was so happy when he discovered the theorem that he offered a sacrifice of oxen. The later discovery that the square root of 2 is irrational and therefore, cannot be expressed as a ratio of two integers, greatly troubled Pythagoras and his followers. They were devout in their belief that any two lengths were integral multiples of some unit length. Many attempts were made to suppress the knowledge that the square root of 2 is irrational. It is even said that the man who divulged the secret was drowned at sea.The Pythagorean Theorem is a statement about triangles containing a right angle. The Pythagorean Theorem states that: ***"The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides."***

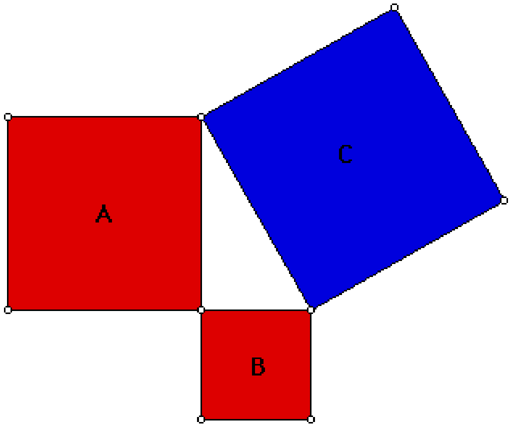


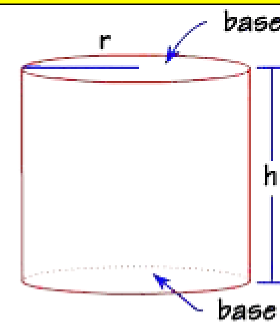
Figure 1

1. Give students opportunity to physically prove the Pythagorean Theorem by manipulating the spaces. See “Discover and Prove an Ancient Theorem” at http://middlemathccss.wordpress.com/8th-grade-math/8th-d4-geometry/showexplainidentify-angle-theorems-angle-sum-exterior-angle-angle-pairs-in-parallel-lines-cut-by-transversal/)
2. Then the equation $a^2 + b^2 = c^2$ will have meaning (square a + square b is indeed equal to square c).
3. Students can then use the theorem to prove the converse (prove perpendicular sides or right triangle) and to solve real-world problems.
4. Another link: http://www.mathsisfun.com/pythagoras.html

Distances with the Pythagorean Theorem

NOTE: Students should use the Pythagorean theorem to find distances on the coordinate grid—this provides background knowledge for connection to the distance formula. If desired or if an honors section, the teacher may wish to have students derive the distance formula, thus connecting it to the Pythagorean Theorem.

Solve Real-World Math Problems involving Volume of Cylinders, Cones, Spheres



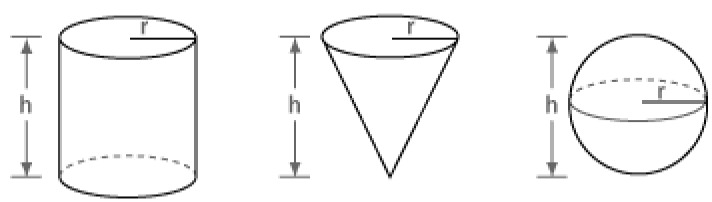
THE CYLINDER

1. Volume is 3-dimensional space
2. As with prisms or cubes, cylinders are right objects that have parallel bases. If we find the area of the base (2D space, but think of this as a layer 1 unit high—so the 2D space becomes 3D and our measure is units^2). We can then multiply by the height (think of the height as layers of the 1unit high base).
3. In this case, the base is a circle, so we find the area of the circle ($A=\pi r^2$), then multiply by the height.
4. Cylinder volume = Bh (B refers to a the 2D area of the base as opposed to b which refers to a 1D length)

CONES and SPHERES (and their relationship to the cylinder):

Note: As states in the core, students can use formulas. To empower their memory and ready access to the formulas it is recommended that they experience the relationship, either visually or via an online interactive.

1. Have students predict the relationship between a cone, a sphere and a cylinder with the same height and circle radius.
2. If you have hollow plastic models, use rice or water—pour it from the cone or the sphere into the cylinder.
3. If you don't, then use this online interactive—it's cool!
   http://www.learner.org/courses/learningmath/measurement/session8/part_b/cylinders.html
4. The cone is 1/3 the volume of the cylinder. The cylinder is filled with 3 cones. Cone volume= 1/3Bh or $1/3(\pi r^2)h$
5. The sphere takes up 2/3 of the cylinder. Two cones will fill up the sphere.
6. A great question to investigate, especially for honors sections: Why **doesn't** the formula "sphere volume= 2/3Bh" work? (does a sphere have a base or height—it only has a diameter). Consider the height of the sphere as twice the radius, then we have $V=(2/3)(\pi r^2)(2r)$ or $V=(4/3)(\pi r^3)$.



7. Extend the discussion: How are pyramids related to prisms of the same base and height?